Nonlinear Wave Interaction of Three Stokes' Waves in Deep Water: Banach Fixed Point Method

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Based on Banach fixed point theorem, a method to calculate nonlinear superposition for three interacting Stokes' waves is proposed in this paper. A mathematical formulation for the nonlinear superposition in deep water and some numerical solutions were investigated. The authors carried out the numerical study with three progressive linear potentials of different wave numbers and succeeded in solving the nonlinear wave profiles of their three wave-interaction, that is, using only linear wave potentials, it was possible to realize the corresponding nonlinear interacting wave profiles through iteration of the method. The stability of the method for the three interacting Stokes' waves was analyzed. The calculation results, together with Fourier transform, revealed that the iteration made it possible to predict higher-order nonlinear frequencies for three Stokes' waves' interaction. The proposed method has a very fast convergence rate.

Key Words: Banach Fixed Point Theorem, Iterative Method, Nonlinear Superposition, Three Stokes' Waves, the Stability of the Method

Nomenclature

a_i : The wave amplitude
B : Bernoulli's Operator
g : The gravity acceleration
k_i : The wave number
P_a : The pressure of the atmosphere
ρ : The constant fluid density
η_{PB} : The free-surface of the perturbation solution
η_n : The free-surface of the iterative method
$\|\cdot\|_\infty$: The sup-norm
ω_i : The wave frequency
Ω_i : Angle measured from positive x-axis

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1. Introduction

A considerable number of studies have been made on the nonlinear water wave profiles. Most of the nonlinear water wave profiles has been treated based on the perturbation theory and CFD (Computational Fluid Dynamics) methods ever since the appearance of Stokes' nonlinear wave theory (Stokes, 1847). For example, computational and theoretical studies of the nonlinear wave profiles could be found (Tsai et al., 1996; Dias and Kharif, 1999; Clamond, Grue, 2001;

Nicholls, Reitich, 2001). On the other hand, there has been a fixed point approach to water wave problem. The fixed point theorem has been successfully applied to many fields of engineering and science to deal with nonlinear problems. One example of the application of the fixed point theorem to the wave problem is given in Bona and Bose (Bona and Bose, 1974). They examined the question of the existence of solitary wave solutions to simple one-dimensional models for long waves in nonlinear dispersive systems.

Recently, with the help of the fixed point theorem, Jang and Kwon (2005) proposed an iterative method to estimate nonlinear wave profiles for one wave train of progressive wave, which was quite different from the traditional perturbation method for nonlinear wave profiles.(Jang and Kwon, 2005 ; Jang et al., 2005(a); 2005(b); 2005(c); 2006(a); 2006(b)). The prsent paper is aimed at an application of the method to the wave interaction problem, that is, the nonlinear interaction of three Stokes' waves in deep water. Only a few studies have been done on nonlinear interacting wave profiles for more than two Stokes' waves. Pierson (1993) has studied about perturbation solutions for sums of interacting Stokes' waves in deep water. However, the derivation of high-order solutions is too lengthy to proceed, even when three Stokes' waves are considered to be summed.

The advantage of the present study over the traditional perturbation approach, which becomes very complicated when the order of nonlinearity increases (Pierson, 1993), is the efficiency of the formulation. There is no need for lengthy derivations with respect to small parameters to construct a series of linearized equations from the nonlinear free surface boundary conditions, as is the case with the perturbation approach. Once the construction of the operator, which has a fixed point, is accomplished, obtaining the solution is straightforward. Simple iterative procedures with a nonlinear operator will suffice. It was concluded that the proposed method, which is based on nonlinear contraction mapping, was a very powerful tool to realize nonlinear superposition of three Stokes' wave profiles in a very easy manner in terms of programming. It was demonstrated that the proposed method gives very accurate numerical results for the nonlinear superposition when compared with those of Pierson (1993): the present study succeeded in finding nonlinear higher-order Fourier's frequencies, which could not have been found in the Pierson's perturbation solution of second order. The importance of multi-component wave interaction can be stated as follows. There is no such thing as a monochromatic wave in the real ocean. A good example of non-monochromatic wave is a confused sea. Even in the case of swell many frequency components of wave co-exist. Therefore when it comes to the real ocean we need to consider many components of wave. The natural step to simulate real ocean is to develop a theory to take into account of many component interaction more than two.

In the study, we began with the mathematical formulation of three interacting Stokes' wave profiles in section 2.

The detailed construction of the operator in the case of deep water is shown in Section 3. The Bernoulli's equation has been interpreted as a nonlinear operator with respect to the free surface elevation. It is proved that the nonlinear operator constructed from Bernoulli's Equation is a contraction mapping (Deimling, 1985). The numerical results including FFT analysis for several wave slopes are shown in Section 4. To test the efficiency of the proposed method, the results were compared with Pierson's (1993) wave profiles of three interacting Stokes' solutions. Numerical convergence tests were conducted based on sup-norm errors to examine the characteristics of the numerical convergences for the iterative solutions. The comparison revealed that the results showed quite a good agreement with each other. The rate of convergence for the proposed operator was also very fast. As a result, all the computations achieved convergence in less than 10 iterations with respect to a specific tolerance.

2. Fixed Point Approach to Three Interacting Stokes' Wave Profile

The amount of fluid is assumed to be homogeneous and the fluid itself incompressible and inviscid. In addition, the fluid motion is irrotational, such that a velocity potential function exists. Suppose that we consider a free surface flow. A Cartesian coordinate system (x, y, z) is adopted, with $z=0$ the plane of the undisturbed free surface and the z-axis positive upwards. We shall restrict the discussion to plane harmonic waves that travel in the x-axis. We begin with three linear progressive wave potentials in deep water with different wave numbers $k_i>0$ for $i=1,2,3$ and consider their linear superposition φ_{sum} :

$$\varphi_{sum}=\varphi_1+\varphi_2+\varphi_3 \text{ where } \varphi_i(\theta_i,\ z) =\frac{a_i g}{\omega_1} e^{k_i z}\sin\theta_i \quad (i=1,2,3) \tag{1}$$

where g, a_i, and ω_i represent the gravity acceleration, the wave amplitude and frequency, respectively. The product $k_i a_i$, the wave slope, is assumed small for $i=1,2,3$. The symbol θ_i denotes the phase function of progressive waves, that is, $k_i x-\omega_i t$ for $i=1,2,3$. The linear dispersion relation in deep water, $\omega_i^2=gk_i$ for $i=1,2,3$ is assumed. The vertical elevation of any point on the free surface of nonlinear superposition of three Stokes' waves may be defined as a function $z=\eta(x, t)$. The surface tension being negligible, then, by applying Bernoulli's equation to the free surface it becomes

$$\frac{\partial\varphi_{sum}}{\partial t}\frac{1}{2}\nabla\varphi_{sum}\cdot\nabla\varphi_{sum}+\frac{P_a}{\rho}+g\eta\approx f(t) \tag{2}$$

where $f(t)$, φ, P_a and ρ stand for Bernoulli's constant, the velocity potential, the pressure of the atmosphere, and the constant fluid density, respectively. Taking Bernoulli's constant $f(t)=P_a/\rho$, we have the approximate expression for the free surface for the three Stokes' wave interaction as

$$\eta\approx-\frac{1}{g}\left[\frac{\partial\varphi_{sum}}{\partial t}+\frac{1}{2}\nabla\varphi_{sum}\cdot\nabla\varphi_{sum}\right]\Bigg|_{z=\eta} \tag{3}$$

The right-hand side of (3) may be viewed as an operator for the free surface for wave interaction, in such a way that we can define a new operator B. We shall call B a Bernoulli's operator in this study :

$$\mathrm{B}(\eta)=-\frac{1}{g}\left[\frac{\partial\varphi_{sum}}{\partial t}+\frac{1}{2}\nabla\varphi_{sum}\cdot\nabla\varphi_{sum}\right]\Bigg|_{z=\eta} \tag{4}$$

Then Bernoulli's operator B can be easily confirmed to be nonlinear, and (3) can simply be written as

$$\eta=\mathrm{B}(\eta) \tag{5}$$

As shown in (4), wave profile η is invariant under the nonlinear operator B : it is called a Banach fixed point (or function), η. If the operator, B, satisfies the following inequality for some constant $\beta<1$:

(6)

then B is considered a contraction mapping. Wave profiles of interaction for three Stokes' waves are realized as the limit of the following sequence

$$\eta_{k+1}=\mathrm{B}(\eta_k) \tag{7}$$

with the initial condition for zero function $\eta_0=0$ (Roman, 1975).

3. Construction of Bernoulli Operator

Substitution of (1) into (4) and calculation yield the explicit form of Bernoulli's operator as follows :

$$\begin{aligned}\mathrm{B}(\eta)=&a_1e^{k_1\eta}\cos\theta_1-\frac{1}{2}k_1a_1^2e^{2k_1\eta}+a_2e^{k_2\eta}\cos\theta_2\\&-\frac{1}{2}k_2a_2^2e^{2k_2\eta}+a_3e^{k_3\eta}\cos\theta_3-\frac{1}{2}k_3a_3^2e^{2k_3\eta}\\&-a_1a_2\sqrt{k_1k_2}\,e^{(k_1+k_2)\eta}\cos(\theta_1-\theta_2)\\&-a_1a_3\sqrt{k_1k_3}\,e^{(k_1+k_3)\eta}\cos(\theta_1-\theta_3)\\&-a_2a_3\sqrt{k_2k_3}\,e^{(k_2+k_3)\eta}\cos(\theta_2-\theta_3)\end{aligned} \tag{8}$$

(8) is based on the three linear potentials of periodic plane waves on deep water. If we set $a_2=a_3=0$, Bernoulli's operator would be reduced to only the sum of the first two terms, that is, $a_1e^{k_1\eta}\cos\theta_1-1/2\,k_1a_1^2e^{2k_1\eta}$, as is the case of Jang and Kwon (2005) for a single gravity wave train. The terms in (8), $a_ie^{k_2\eta}\cos\theta_i-(1/2)\,k_ia_i^2e^{2k_i\eta}$, $(i=1,2,3)$ represent a Bernoulli operator due to a single wave train φ_i in (1). The last three terms $-a_ia_j\sqrt{k_ik_j}e^{(k_i+k_j)\eta}\cos(\theta_i-\theta_j)$ are closely related with a nonlinear interacting wave profile between two trains of gravity waves φ_i and $\varphi_j(i,j=1,2,3)$ in (1).

If we substitute (8) into (6) and employ triangle inequality and smallness of wave slopes, then we arrive at

$$\beta = k_1 a_1(1+k_1 a_1) + k_2 a_2(1+k_2 a_2) + k_3 a_3(1+k a_3) + a_1 a_2(k_1+k_2)\sqrt{k_1 k_2} + a_1 a_3(k_1+k_3)\sqrt{k_1 k_3} + a_2 a_3(k_2+k_3)\sqrt{k_2 k_3} \quad (9)$$

Here the constant β is known as the contraction coefficient β (Zeidler, 1986). In order for the iterative scheme to converge, we need a criterion, that is, the contraction condition, $\beta < 1$:

$$k_1 a_1(1+k_1 a_1) + k_2 a_2(1+k_2 a_2) + k_3 a_3(1+k a_3) + a_1 a_2(k_1+k_2)\sqrt{k_1 k_2} + a_1 a_3(k_1+k_3)\sqrt{k_1 k_3} + a_2 a_3(k_2+k_3)\sqrt{k_2 k_3} < 1 \quad (10)$$

If wave conditions such as wave numbers and amplitudes are satisfied with the inequality (10), then the iteration scheme (7) works and yields wave profiles of three interacting Stokes' waves. The existence and uniqueness of the solution for the interacting wave profile was shown by Jang et al.(2006(b)). (9) illustrates to us that the value of the contraction coefficient tends to increase as wave slopes become large. Therefore, the contraction coefficient can be thought of as a measure of nonlineararity for interacting wave profiles.

The contraction condition, $\beta < 1$ in equality (10) guarantees that the mapping B is contraction. In order to satisfy the fixed point theorem, we need to satisfy two additional conditions : Solution space is a complete space on which the mapping B is defined should be chosen, and the mapping B maps the space into itself. First, for a fixed time, any continuous real valued function can be defined for the Bernoulli operator in Eq. (8): i.e., it is possible for the Bernoulli operator in Eq. (8) to have domain space of continuous functions. Therefore, it is natural to introduce a solution space X of continuous functions (to which the interacting wave profiles η belong) with an usual topology : we introduce the uniform metric or sup-norm in this paper (Roman, 1975). Then the solution space X of continuous functions equipped with the uniform metric or sup-norm- topology is well known as complete (Roman, 1975). Second, from calculus, it is known that the multiplication, subtraction, and composition operations of continuous functions yields a continuous function. Therefore, the mapping of the Bernoulli operator in Eq. (8) maps X into itself, because the mapping B involves the multiplication, subtraction, and composition operations. Therefore, all the three conditions for the fixed point theorem are satisfied for the mapping B in this paper.

4. Numerical Results

In this section, we will present the numerical results of wave profiles of three interacting Stokes' waves. For solutions of the wave profile, (7) is iterated with an initial condition for zero function of mean water level (for $n=0$), that is, $\eta_0=0$. For comparison purposes, the three interacting Stokes' wave profiles of perturbation second order solution suggested by Pierson (1993) was employed. For numerical study, we examine four different wave profiles. Their wave information is tabulated in Table 1 and 2. The contraction coefficients corresponding to the wave-parameters are also shown in Table 1 and 2. Since they are less than 1, our iteration scheme (7) is known to converge. (Roman, 1975)

Case 1 (Table 1) represents a mild slope and case 4 (Table 1) the other extreme. For the numerical convergence test, we calculate the fol-

Table 1 Various Wave-Parameters investigated

Case Number	k_1	a_1	k_2	a_2	k_3	a_3	β
Case 1	0.3	0.05	0.4	0.05	0.5	0.05	0.0636
Case 2	0.3	0.10	0.4	0.10	0.5	0.10	0.1345
Case 3	0.3	0.15	0.4	0.15	0.5	0.15	0.2127
Case 4	0.3	0.22	0.4	0.22	0.5	0.22	0.3161

Table 2 Various Wave-Parameters for amplitude variations

Case Number	k_1	a_1	k_2	a_2	k_3	a_3	β
Case 1	0.3	0.07	0.4	0.10	0.5	0.16	0.1610
Case 2	0.3	0.08	0.4	0.15	0.5	0.25	0.2530
Case 3	0.3	0.10	0.4	0.18	0.5	0.30	0.3160

lowing coefficients, which are norm errors,

$$\mu_\infty = \frac{\| \eta_{PB} - \eta_n \|_\infty}{\| \eta_{PB} \|_\infty} \tag{11}$$

as a function of the number of iterations n. The notation $\|\cdot\|_\infty$ represents the sup-norm. Here the coefficient is normalized by the norm of the Pierson (1993) perturbation solution of the second order, denoted by η_{PB}.

For the four different wave-parameter cases, the coefficient values for μ_∞ are plotted in Fig. 1, which are presented in terms of the number of iterations. Figure 1 shows that our solution strategy η_n is converging to the nonlinear perturbation solution η_{PB}, regardless of all the wave-parameter cases. From this analysis, it is clear that the norm error μ_∞ is reduced as the number of iterations n increases and that their convergence rates are very high. We confirmed all of the cases converged within less than 10 iterations.

In Fig. 2 the process by which the wave profiles change is shown ($t=0$). The change is shown in terms of the number of iterations. In Fig. 3, the obtained converged solutions ($t=0$) are compared with the corresponding linear and nonlinear second-order wave profiles as suggested by Pierson (1993). Fig. 3 shows an excellent agreement between the results obtained by the proposed scheme and those of Pierson. Time evolutions of wave propagation are illustrated for the case 4 (Table 1) and 3 (Table 2) in Fig. 5. It is not difficult to observe dispersive waves are propagating to positive x-axis.

To examine the nonlinear behavior of the solution, the Fourier transform was introduced. The amplitude spectra of the solutions are presented in Fig. 4. To highlight the peaks in the amplitude spectrum, linear-log coordinates are illu-

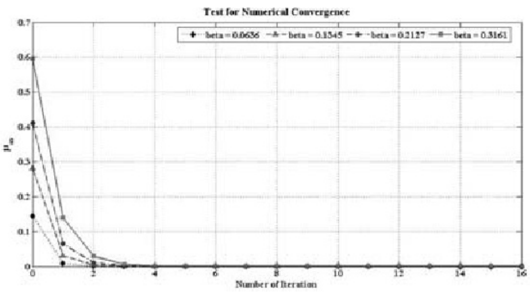
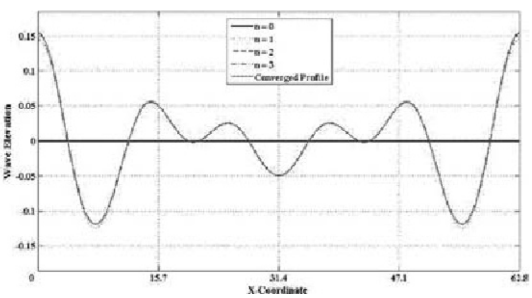
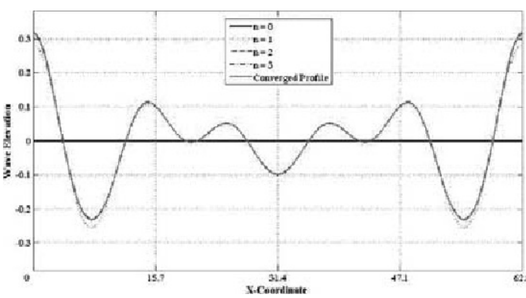


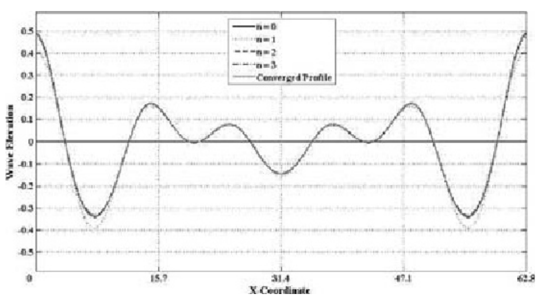
Fig. 1 Test for numerical convergence using the sup-norm



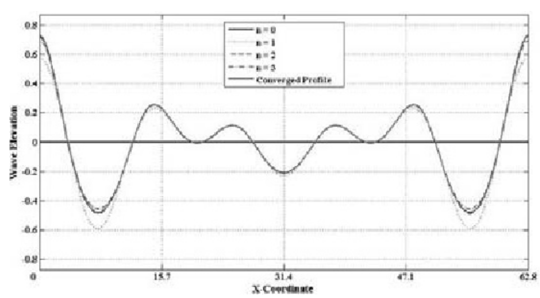
(a) Case 1 (Table 1): $\beta=0.0636$



(b) Case 2 (Table 1): $\beta=0.1345$



(c) Case 3 (Table 1): $\beta=0.2127$

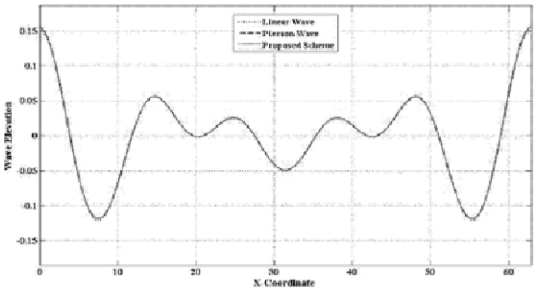


(d) Case 4 (Table 1): $\beta=0.3161$

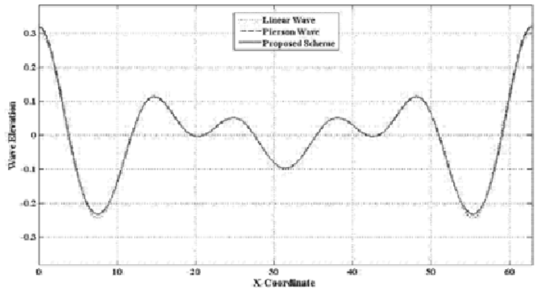
Fig. 2 Convergence behavior of η_n

strated. For the four cases, three dominant peaks at $k_1(k=0.3)$, $k_2(k=0.4)$ and $k_3(k=0.5)$ appear clear as is expected. They are the fundamental wave number components in this study.

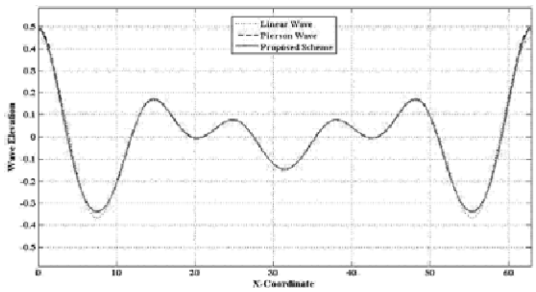
In Fig. 4, we can see the peaks of the proposed scheme due to the double wave number components at $2k_1(k=0.6)$, $2k_2(k=0.8)$ and $2k_3(k=1.0)$, the sum wave numbers at $k_1+k_2(k=0.7)$, $k_1+k_3(k=0.8)$ and $k_2+k_3(k=0.9)$ even though their magnitudes are very small compared to those of the fundamental wave number components. When the peaks of the proposed scheme are com-



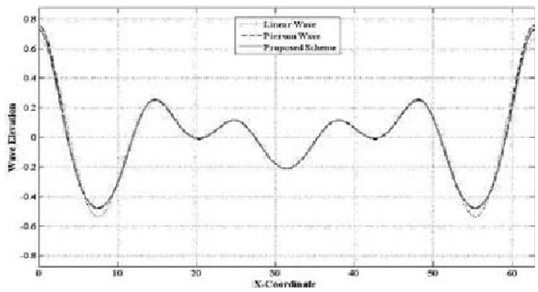
(a) Case 1 (Table 1): $\beta=0.0636$



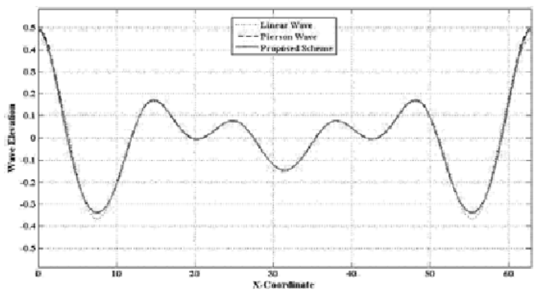
(b) Case 2 (Table 1): $\beta=0.1345$



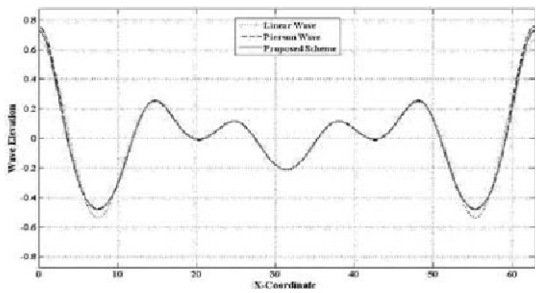
(c) Case 3 (Table 1): $\beta=0.2127$



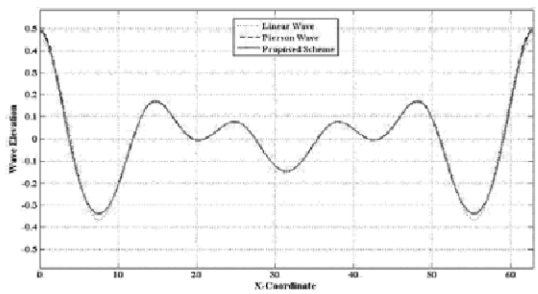
(d) Case 4 (Table 1): $\beta = 0.3161$



(e) Case 1 (Table 2): $\beta=0.1610$



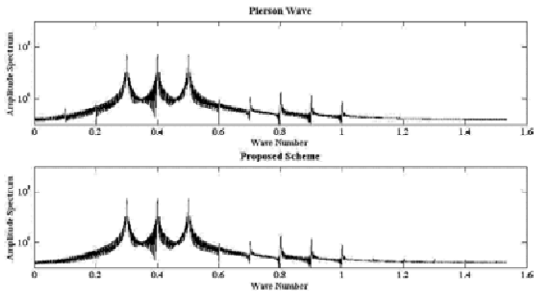
(f) Case 2 (Table 2): $\beta=0.2530$



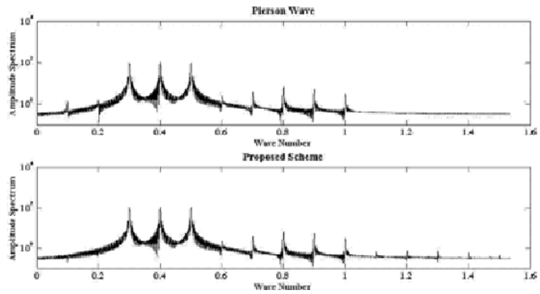
(g) Case 3 (Table 2): $\beta=0.3160$

Fig. 3 Comparison of Wave Profiles

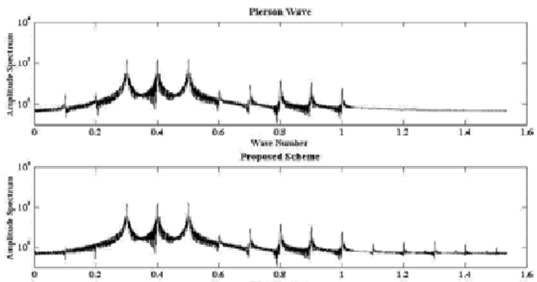
pared with those of the Pierson's solution, they are in good agreement with each other for all four



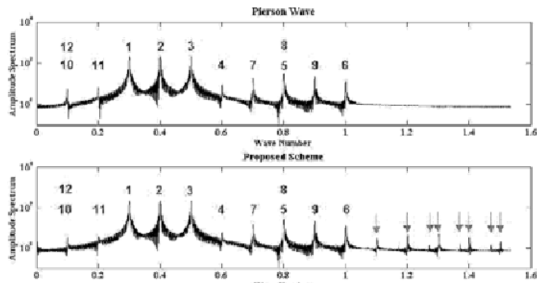
(a) Case 1 (Table 1): $\beta=0.0636$



(b) Case 2 (Table 1) : $\beta=0.1345$

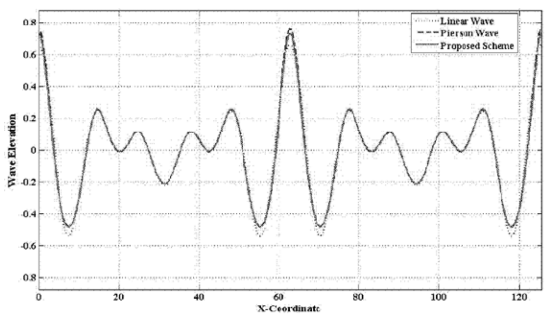


(c) Case 3 (Table 1): $\beta=0.2127$

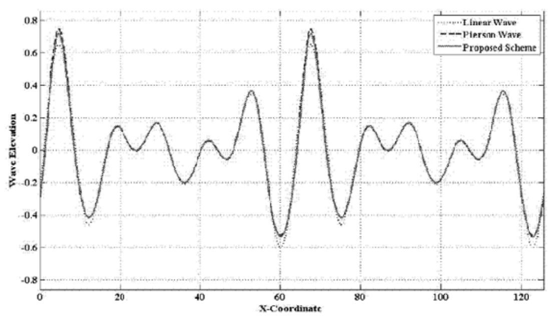


(d) Case 4 (Table 1): $\beta=0.3161$

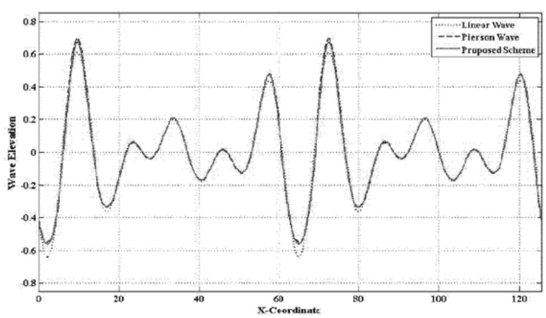
Fig. 4 Comparison of Wave Amplitude Spectra



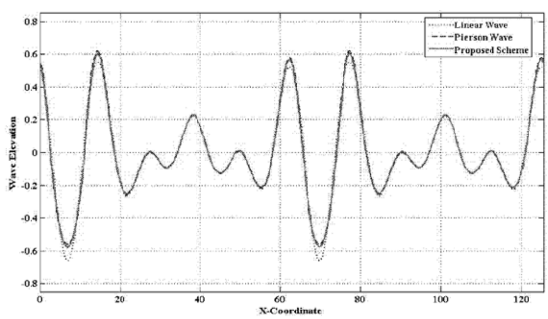
t=0



t=1



t=2



t=3

(a) Case 4 (Table 1)

Fig. 5 Time evolution of wave profiles cases.

However, in Fig. 4(a), it may be hard to observe the peaks due to the difference wave numbers at $k_2-k_1(k=0.1)$, $k_3-k_1(k=0.2)$ and $k_3-k_2(k=0.1)$ in the amplitude spectrum : the hidden peaks are recovered when the contraction coefficients are relatively larger cases as shown in Fig. 4(b), (c) and (d). The interacting components are illustrated in Table 3. The numbers corresponding to the frequency components as shown in Table 5 are written in Fig. 4(d).

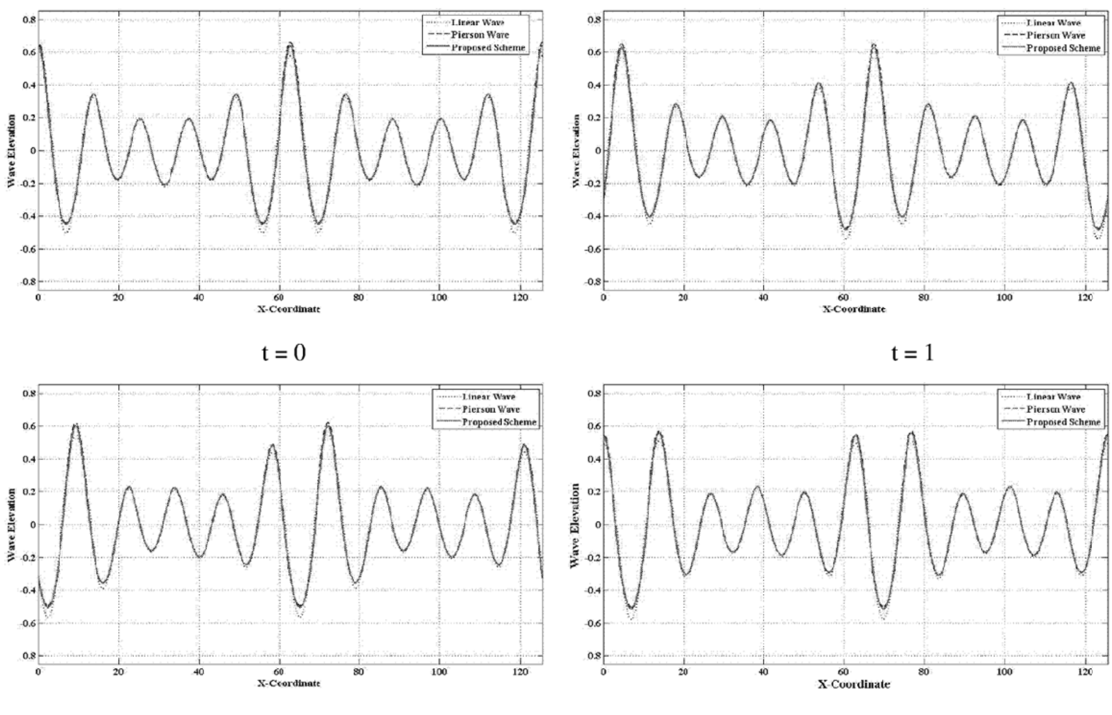
An interesting phenomenon was observed : the proposed scheme yields peaks at higher frequencies, as shown in red-colored arrows in Fig. 4 (d). Their existence cannot be explained by the Pierson's perturbation solution of second order. They may be corresponding to higher order nonlinear frequencies for three interacting Stokes' waves, which should be investigated further.

Table 3 Various Wave-Parameters investigated (effect of wave numbers)

Case Number	k_1	a_1	k_2	a_2	k_3	a_3	β
Case 1	0.1	0.05	0.2	0.06	0.3	0.07	0.0395
Case 2	0.3	0.05	0.4	0.06	0.5	0.07	0.0795
Case 3	0.8	0.05	1.3	0.06	1.5	0.07	0.2734

To investigate the effect of a variation of wave numbers, we tabulate wave parameters in Table 3. With the data of the parameters, we calculate interacting wave profiles and their FFT results as shown in Fig. 6. It is interesting to find that strong nonlinear interaction (Case 3) is observed in the zone of high wave numbers when compared to Case 1.(Case 3 corresponds to larger wave number)

Finally, let us add some calculation about two dimensional calculations of the dispersive waves through the iteration method in Eq. (7). At first, it is needed to introduce the vector wavenumber $\vec{k}_i=[(k_x)_i, (k_y)_i]$ in order to describe propagating directionality of waves :



t = 0

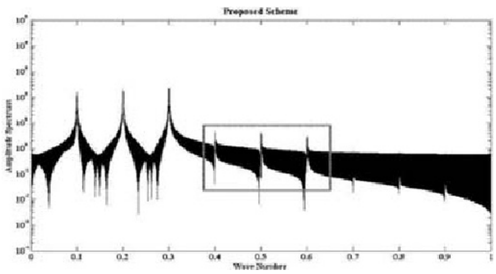
t = 1

t = 2

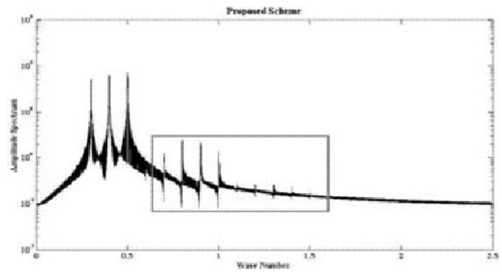
t = 3

(b) Case 3 (Table 2)

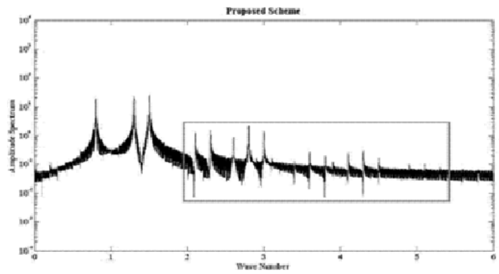
Fig. 5 Time evolution of wave profiles



(a) Case 1 (Table 3)

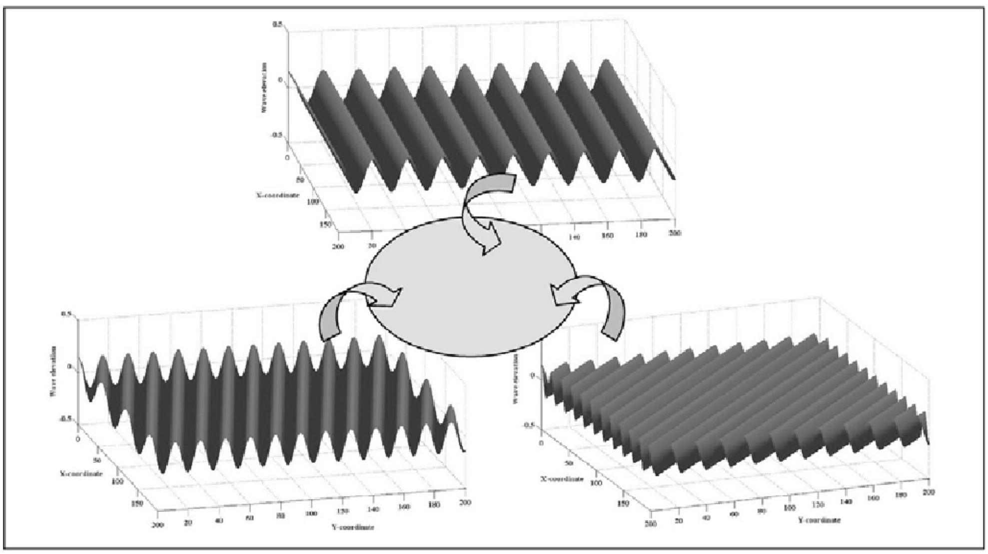


(b) Case 2 (Table 3)

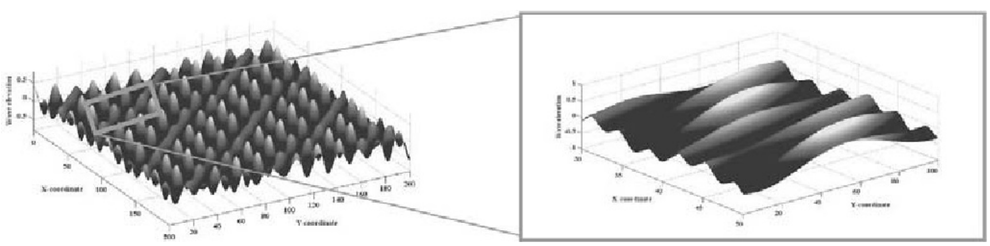


(c) Case 3 (Table 3)

Fig. 6 Amplitude spectra (effect of wave numbers)



(a) Three monochromatic directional waves



(b) The result of the three interacting waves

Fig. 7 Illustration of 2D plane wave calculation

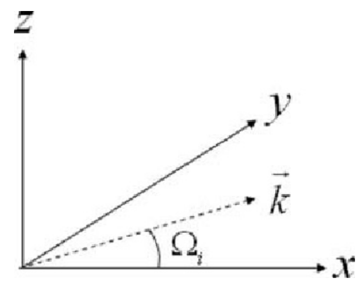


Fig. 8 Definition sketch for vector wavenumber $\vec{k}$

$$k_x=|\vec{k}|\cos\Omega,\ k_y=|\vec{k}|\sin\Omega$$

where the symbol Ω_i denotes an angle measured from positive x-axis as shown in Fig. 8.

Then the phase function in Eq. (1) takes the form

$$\theta_i(x,y,t)=(k_x)_i\cos\Omega_i+(k_y y\sin\Omega_i-\omega_i t) \quad (i = 1, 2, 3)$$

Wave parameters for two dimensional waves are tabulated in Table 4. And their two dimensional numerical calculations are depicted in Fig. 7.

Table 4 Wave-Parameters for two dimensional waves

$\|\vec{k}_1\|$	a_1	Ω_1 (deg.)	$\|\vec{k}_2\|$	a_2	Ω_2 (deg.)	$\|\vec{k}_3\|$	a_3	Ω_3 (deg.)
0.3	0.15	0	0.4	0.15	15	0.5	0.15	30

Table 5 Peaked frequencies in amplitude spectra

Indicated Number	Frequency Components	Numerical Value of Frequency
1	k_1	0.3
2	k_2	0.4
3	k_3	0.5
4	$2k_1$	0.6
5	$2k_2$	0.8
6	$2k_3$	1.0
7	k_1+k_2	0.7
8	k_1+k_3	0.8
9	k_2+k_3	0.9
10	k_2-k_1	0.1
11	k_3-k_1	0.2
12	k_3-k_2	0.1

5. Concluding remarks

By applying Banach fixed-point theorem to Bernoulli's Equation, we have proposed a method to realize an interacting wave profile for three Stokes' waves in deep water. The formulation and process of the computation involved are very handy even though three Stokes' wave interactions are taken into account. This is a completely different point of view when compared to the perturbation approach of Pierson (1993). The numerical results of the method are compared with those of Pierson's Formulation. The comparison revealed that the results showed quite a good agreement with each other. It is interesting that the iteration, based on linear progressive potential solutions, enabled us to observe the higher-order nonlinear frequencies for three interacting Stokes' waves that Pierson's second order solution could not predict.

In conclusion, we have proposed a method, which was based on nonlinear contraction mapping and was proved to be a very powerful tool to estimate a nonlinear interaction for three Stokes' wave profiles in a very easy manner.

Acknowledgements

This work was supported by Korea Science and Engineering Foundation through the Advanced Ship Engineering Research Center at Pusan National University.

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